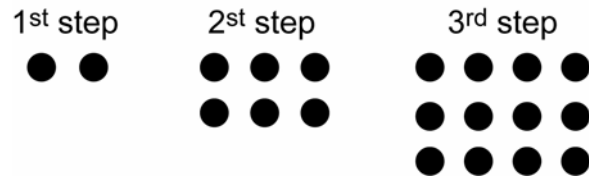**Grade 7 – Algebraic Relationships**
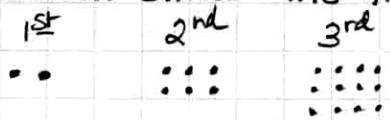
Pattern Of Dots

A pattern of dots is shown. At each step, more dots are added to the pattern. The pattern continues infinitely.

1st step   2st step   3rd step

● ●   ● ● ●   ● ● ● ●
              ● ● ●   ● ● ● ●
                             ● ● ● ●

How many dots are in the 20th step?

The problem shows the first three steps in a pattern.

1st   2nd   3rd

• •   ∶ ∶   ∶ ∶∶

The question wants you to find the twentieth step in the pattern of dots

I will take the steps in the dot pattern and convert them into a table using x to represent the step number and using y to represent the number of dots in the step.

The pattern I used was $x(x+1)$ to get y.

Example:
$(3, y)$
$3(3+1) = 3(4) = 12$
So step 3 has 12 dots

x	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
y	2	6	12	20	30	42	56	72	90	110	132	156	182	210	240	272

x	17	18	19	20
y	306	342	380	420

(420 dots)

Verification

the line of the graph grows so it doesn't have a constant rate of change, but a relative one:

step (x)	(x+1)	# of dots x(x+1)	
1	2	2	
			> 4
2	3	6	
			> 6
3	4	12	
			> 8
4	5	20	
			> 10
5	6	30	
			> 12
6	7	42	
			> 14
7	8	56	
			> 16
8	9	72	
			> 18
9	10	90	
			> 20
10	11	110	
			> 22
11	12	132	
			> 24
12	13	156	
			> 26
13	14	182	
			> 28
14	15	210	
			> 30
15	16	240	
			> 32
16	17	272	
			> 34
17	18	306	
			> 36
18	19	342	
			> 38
19	20	380	
			> 40
20	21	420	

2 – Algebraic Relationships
Pattern of dots
A pattern of dots is shown. At each step, more dots are added to the pattern.
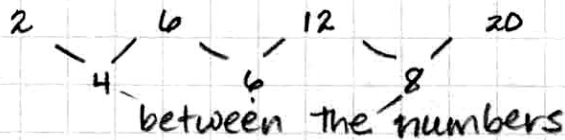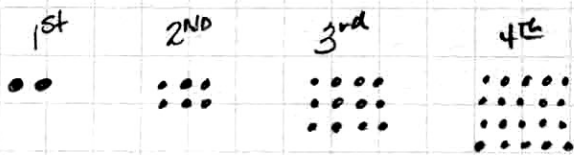The pattern continues infinitely.

1st step   2st step   3rd step

How many dots are in the 20th step?

<u>7 – 2 – 1</u>

CU   6   The translation of the key concepts (identification of a pattern and extending the pattern) is enhanced with the details provided in the description of the pattern, the generalization of the pattern and the description of what the graph would look like.

PS   5   The process of converting the number of dots at each step into a table, writing a generalization for finding the number of dots (y) when given the step (x), and then carrying out the pattern to the 20th place is thoroughly developed. [The process might have been enhanced if s/he had used the generalization $x(x + 1)$ to arrive at the same number of dots $20\,(21) = 420$ as s/he had when completing the table.]

V   5   The verification includes a complete solution to the task – the generalization of $x\,(x + 1)$ carried out until the step number was 20, resulting in a table of growing differences as described in the prose – is thoroughly developed.

C   5   The path connecting the restating of the task to the process of finding a generalization for the pattern to completing the pattern to the review showing the growing differences all towards the identified solution is well organized and detailed making it thoroughly developed.

Acc.   5   420 dots in the 20th step is a mathematically justifiable solution supported by the work.

HOW MANY DOTS ARE IN THE 20TH STEP?

1st	2ND	3rd	4th

•• ∴∴ (dot patterns)

2   6   12   20

4   6   8

between the numbers

Keep adding 2 until you get to the 20ᵗʰ step

Step	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
How many	2	6	12	20	30	42	56	72	90	110	132	156	182	210	240	272	306	342	380	420

4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40

There would be 420 dots on the 20ᵗʰ step. I figured that out by adding 2 to all the differences and then adding that total to the number before (ex. $\overset{3\quad 6}{\underset{3}{\vee}}$ 6+3=9)

[420 DOTS]

VERIFICATION

$2+4+6+8+10+12+14+16+18+20+22+24+26+28+30+32+34+36$
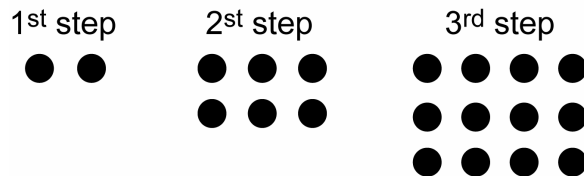$+ 38+40 = 420$

I ADDED ALL OF THE #'S STARTING WITH 2 UNTIL I GOT TO STEP 20.

[420 DOTS]

2 – Algebraic Relationships

Pattern of dots
A pattern of dots is shown. At each step, more dots are added to the pattern.
The pattern continues infinitely.
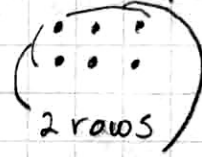

1st step   2st step   3rd step
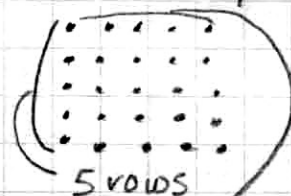
How many dots are in the 20th step?

<u>7 – 2 – 2</u>

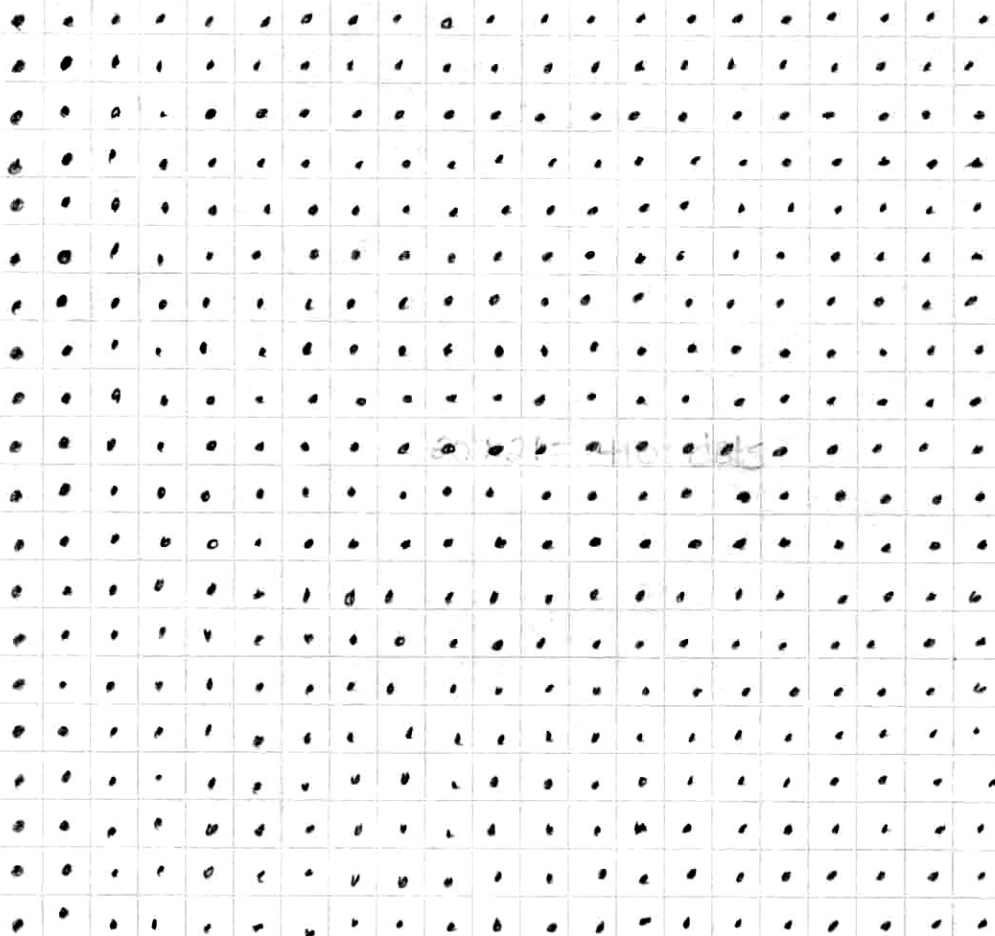|		
CU	4	The translation of the key concepts (identification of a pattern and extending the pattern) is completed.
PS	5	The process of creating a table with the steps in one row and the number of dots in the other, then finding the pattern of growth along the table and continuing it until the 20th step is completed. The details added in the prose and the example make the process thoroughly developed.
V	3	Finding the sum of the differences growing by 2's "until I got to step 20" relied on the original solution for the identification of the pattern. Without verifying the pattern, the review is only partially completed.
C	4	The path connecting the translation of the task to the process of creating and completing the table to the verification all towards the identified solution is complete.
Acc.	5	420 dots in the 20th step is a mathematically justifiable solution supported by the work.

1ˢᵗ step

(1 row)

1+1 = 2 colums

1·2 = 2 dots

2ⁿᵈ step

2 raws

2+1 = 3 colums

2·3 = 6 dots

3ʳᵈ step

3 rows

3+1 = 4 colums

3·4 = 12 dots

4ᵗʰ step

5 rows

4+1 = 5 colums

5·5 = 25 dots

20th step

proof:
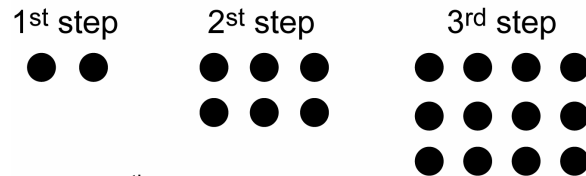
20 × 21 = 410 dots

When I made the dots I counted 420 dots

2 – Algebraic Relationships

Pattern of dots
A pattern of dots is shown. At each step, more dots are added to the pattern.
The pattern continues infinitely.

1st step   2st step   3rd step

How many dots are in the 20th step?

7 – 2 – 3

CU   4   The translation of the key concepts (identification of a pattern and extending the pattern) is complete when s/he is able to draw a 20 row and 21 column array and count the dots.

PS   3   The identification of the relationship between the rows and columns falls apart in the 4th step – which should have been 5 columns of 4 rows for 20 dots. S/he could not use a generalization as a process, but could only draw the number of dots and count them – making the process only partially useful.

V   4   The "proof 20 x 21 = 420" does in fact verify the generalization worked for the 20th step and matches the number of dots counted when drawing out the array for the 20th step.

C   3   The path connecting the translation of the task into an identified pattern to the process of making all the dots in a 20 by 21 array is complete. There is a significant gap in connecting the array to the verification which shows a generalization that wasn't previously identified, and didn't fit the results shown in the 4th step.

Acc.   5   420 dots in the 20th step is a mathematically justifiable solution supported by the work.