### Mathematics Claim #4

**MODELING AND DATA ANALYSIS**

Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

### Rationale for Claim #4

“Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decision-making.” (p.72, CCSSM)

As such, modeling is the bridge across the “school math”/”real world” divide that has been missing from many mathematics curricula and assessments. It is the twin of *mathematical literacy*, the focus of the PISA international comparison tests in mathematics. CCSSM features modeling as both a mathematical practice at all grades and a content focus in high school.

“Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.” (Practice 4; CCSSM)

In the real world, problems do not come neatly ‘packaged’. Real world problems are complex, and often contain insufficient or superfluous data. Assessment tasks will involve *formulating* a problem that is tractable using mathematics - that is, formulating a model. This will usually involve making assumptions and simplifications. Students will need to select from the data at hand, or estimate data that are missing. (Such tasks are therefore distinct from the problem-solving tasks described in Claim #2, that are well-

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31 In their everyday life and work, most adults use none of the mathematics they are first taught after age 11. They often do not see the mathematics that they do use (in planning, personal accounting, design, thinking about political issues etc.) as mathematics.
Overview of Claims and Evidence for CCSS Mathematics Assessment

formulated). Students will identify variables in a situation, and construct relationships between these. When students have formulated the problem, they then tackle it, often in a decontextualized form, before interpreting their results and checking them for reasonableness.

“Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.” (Practice 2; CCSSM)

Finally, students interpret, validate and report their solutions through the successive phases of the modeling cycle, illustrated in the following diagram from CCSSM.

Assessment tasks will also test whether students are able to use technology in this process.

“When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.” (Practice 5; CCSSM)

What sufficient evidence looks like for Claim #4

A key feature of items and tasks in Claim #4 is the student is confronted with a contextualized, or “real world” situation and must decide which information is relevant and how to represent it. As some of the examples provided below illustrate, “real world” situations do not necessarily mean questions that a student might really face; it means that mathematical problems are embedded in a practical, application context. In this way, items and tasks in Claim #4 differ from those in Claim #2, because while the goal is clear, the problems themselves are not yet fully formulated (well-posed) in mathematical terms.
Overview of Claims and Evidence for CCSS Mathematics Assessment

Items/tasks in Claim #4 assess student expertise in choosing appropriate content and using it effectively in formulating models of the situations presented and making appropriate inferences from them. Claim #4 items and tasks should sample across the content domains, with many of these involving more than one domain. Items and tasks of this sort require students to apply mathematical concepts at a significantly deeper level of understanding of mathematical content than is expected by Claim #1. Because of the high strategic demand that substantial non-routine tasks present, the technical demand will be lower – normally met by content first taught in earlier grades, consistent with the emphases described under Claim #1. Although most situations faced by students will be embedded in longer performance tasks, within those tasks, some selected response and short constructed response items will be appropriate to use.

**Accessibility and Claim #4:** Many students with disabilities can analyze and create increasingly complex models of real world phenomena but have difficulty communicating their knowledge and skills in these areas. Successful adults with disabilities rely on alternative ways to express their knowledge and skills, including the use of assistive technology to construct shapes or develop explanations via speech to text. Others rely on calculators, physical objects, or tools for constructing shapes to work through their analysis and reasoning process.

For English learners, it will be important to recognize ELL students’ linguistic background and level of proficiency in English in assigning tasks and to allow explanations that include diagrams, tables, graphic representations, and other mathematical representations in addition to text. It will also be important to include in the scoring process a discussion of ways to resolve issues concerning linguistic and cultural factors when interpreting responses.

**Assessment Targets for Claim #4**

Claim #4 is aligned to the mathematical practices from the MCCSS, which are consistent across grade levels. For this reason, the Assessment Targets are not divided into a grade-by-grade description. Rather, a general set of targets is provided, which can be used as guidance for the development of item and test specifications for each grade.

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**SUMMATIVE ASSESSMENT TARGETS**

**Providing Evidence Supporting Claim #4**

**Claim #4 - Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.**

To preserve the focus and coherence of the standards as a whole, tasks must draw clearly on knowledge and skills that are articulated in the content standards. At each grade level, the content standards offer natural and productive settings for generating evidence for Claim #4. Tasks generating evidence for Claim #4 in a given grade will draw upon knowledge and skills articulated in the progression of standards up to that grade, with strong emphasis on the “major” work of the grades.

Any given task will provide evidence for several of the following assessment targets; each of the following...
Overview of Claims and Evidence for CCSS Mathematics Assessment

Targets should not lead to a separate task.

Relevant Verbs for Identifying Content Clusters and/or Standards for Claim #4

“model,” “construct,” “compare,” “investigate,” “build,” “interpret,” “estimate,” “analyze,” “summarize,” “represent,” “solve,” “evaluate,” “extend,” and “apply”

Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace. (DOK 2, 3)
Problems used to assess this target for Claim #4 should not be completely formulated (as they are for the same target in Claim #2), and require students to extract relevant information from within the problem and find missing information through research or the use of reasoned estimates.

Target B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem. (DOK 2, 3, 4).
Tasks used to assess this target include CR9 (“counting trees”) from the assessment sampler, and “design a tent” below.

Target C: State logical assumptions being used. (DOK 1, 2)
Tasks used to assess this target ask students to use stated assumptions, definitions, and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate.

Target D: Interpret results in the context of a situation. (DOK 2, 3)
Tasks used to assess this target should ask students to link their answer(s) back to the problem’s context. (See Claim #2, Target C for further explication.)

Target E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon. (DOK 3, 4)
Tasks used to assess this target ask students to investigate the efficacy of existing models (e.g., develop a way to analyze the claim that a child’s height at age 2 doubled equals his/her adult height) and suggest improvements using their own or provided data.

Other tasks for this target will ask students to develop a model for a particular phenomenon (e.g., analyze the rate of global ice melt over the past several decades and predict what this rate might be in the future). Longer constructed response items and extended performance tasks should be used to assess this target.

Target F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)
Unlike Claim #2 where this target might appear as a separate target of assessment (see Claim #2, Target D), it will be embedded in a larger context for items/tasks in Claim #4. The mapping of relationships should be part of the problem posing and solving related to Claim #4 Targets A, B, E, and G.

Target G: Identify, analyze and synthesize relevant external resources to pose or solve problems.

At the secondary level, these chains should typically take a successful student 10 minutes to complete. Times will be somewhat shorter for younger students, but still giving them time to think and explain. For a minority of these tasks, subtasks may be constructed to facilitate entry and assess student progress towards expertise. Even for such “apprentice tasks” part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes.

Excerpt from: Smarter Balanced Assessment Consortium
Content Specifications for the Summative Assessment of the "Common Core State Standards for Mathematics"
DRAFT TO ACCOMPANY GOVERNING STATE VOTE ON ASSESSMENT CLAIMS
March 20, 2012
Posted by Oregon Dept. of Education, May 2012 - to be updated following future updates from Smarter Balanced
Especially in extended performance tasks (those requiring 1-2 class periods to complete), students should have access to external resources to support their work in posing and solving problems (e.g., finding or constructing a set of data or information to answer a particular question or looking up measurements of a structure to increase precision in an estimate for a scale drawing). Constructed response items should incorporate “hyperlinked” information to provide additional detail (both relevant and extraneous) for solving problems in Claim #4.

**Design a Tent**  
(Grade 8)

Your task is to design a 2-person tent like the one in the picture.

Your design must satisfy these conditions:

- It must be big enough for someone to move around in while kneeling down, and big enough for all their stuff.
- The bottom of the tent will be made from a thick rectangle of plastic.
- The sloping sides and the two ends will be made from a single, large sheet of material.
- Two vertical tent poles will hold the whole tent up.

Make drawings to show how you will cut the plastic and the material.  
Make sure you show the measures of all relevant lengths and angles clearly on your drawings, and explain why you have made the choices you have made.
**The Taxicab Problem (Grade 9)**

You work for a business that has been using two taxicab companies, Company A and Company B.

Your boss gives you a list of (early and late) "Arrival times" for taxicabs from both companies over the past month.

Your job is to analyze those data using charts, diagrams, graphs, or whatever seems best. You are to:

1. Make the best argument that you can in favor of Company A;
2. Make the best argument that you can in favor of Company B;
3. Write a memorandum to your boss that makes a reasoned case for choosing one company or the other, using the relevant mathematical tools at your disposal.

Here are the data:

<table>
<thead>
<tr>
<th>Company A</th>
<th>Company B</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 min. 30 sec. EARLY</td>
<td>2 min. 15 sec. LATE</td>
</tr>
<tr>
<td>45 sec. LATE</td>
<td>3 min. 45 sec. LATE</td>
</tr>
<tr>
<td>1 min. 30 sec. LATE</td>
<td>1 min. 30 sec. LATE</td>
</tr>
<tr>
<td>4 min. 30 sec. LATE</td>
<td>5 min. LATE</td>
</tr>
<tr>
<td>45 sec. EARLY</td>
<td>2 min. 15 sec. LATE</td>
</tr>
<tr>
<td>2 min. 30 sec. EARLY</td>
<td>2 min. 30 sec. LATE</td>
</tr>
<tr>
<td>4 min. 45 sec. LATE</td>
<td>1 min. 15 sec. LATE</td>
</tr>
<tr>
<td>3 min. 45 sec. LATE</td>
<td>45 sec. LATE</td>
</tr>
<tr>
<td>30 sec. LATE</td>
<td>2 min. 45 sec. LATE</td>
</tr>
<tr>
<td>5 min. 30 sec. LATE</td>
<td>3 min. LATE</td>
</tr>
<tr>
<td>1 min. 30 sec. EARLY</td>
<td>30 sec. EARLY</td>
</tr>
</tbody>
</table>

To work this problem the student needs to decide how to conceptualize the data, which computations to make, and how to represent the data from those computations. It turns out that Company A has a better mean arrival time than company B (this is the core of the argument they should make if they decide in favor of A - and for which they would receive credit), but it has a much greater spread of arrival times. The narrow spread is the compelling argument for B - you can’t risk waiting for a cab that is extremely...
late, even if the company’s average is good. Thus the best solution is to use company B, but to ask that they come a bit earlier than you actually need them - thus guaranteeing they arrive on time.\(^{33}\)

With such problems, we see how students decide which information is a given problem context is important, and then how they use it. This is a dimension that is not found in Claim #2.

**Types of Extended Response Tasks for Claim #4**

The following types of tasks, when well-designed and developed through piloting, naturally produce evidence on the aspects of a student’s performance relevant to this claim. Some examples of are given below, with an analysis of what they assess.

**Making decisions from data:** These tasks require students to select from a data source, analyze the data and draw reasonable conclusions from it. This will often result in an evaluation or recommendation. The purpose of these tasks is not to provide a setting for the student to demonstrate a particular data analysis skill (e.g. box-and-whisker plots)—rather, the purpose is the drawing of conclusions in a realistic setting, using a range of techniques.

**Making reasoned estimates:** These tasks require students to make reasonable estimates of things they do know, so that they can then build a chain of reasoning that gives them an estimate of something they do not know.

<table>
<thead>
<tr>
<th>Math – Grade 7</th>
<th>Item Type: CR</th>
<th>DOK: (Webb 1- 4) 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Domain(s):</strong> Geometry</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Content Cluster(s) and/or Standard(s)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.G Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.SP Investigate patterns of association in bivariate data.</td>
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<tr>
<td><strong>Claim #4 Assessment Targets</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Target C: State logical assumptions being used.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Target D: Interpret results in the context of a situation.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{33}\) This problem has been used with thousands of students, and is well within their capacity. It is very different from a problem that gives the students the same numbers and asks them to calculate the mean times, ranges, etc.
Wrap the Mummy

Pam is thirteen today.
She is holding a party at which she plans to play the game 'Wrap the mummy'.
In this game, players try to completely cover themselves with toilet paper.

A roll of toilet paper contains 100 feet of paper, 4 inches wide.
Will one toilet roll be enough to wrap a person?
Describe your reasoning as fully as possible.
(You will need to estimate the average size of an adult person)

Plan and design tasks: Students recognize that this is a problem situation that arises in life and work.
Well-posed planning tasks involving the coordinated analysis of time, space, and cost have already been commended for assessing Claim #2. For Claim #4, the problem will be presented in a more open form, asking the student to identify the variables that need to be taken into account, and the information they will need to find. An example of a relatively complex plan and design task is:

Planning a Class Trip

You and your friends on the Class Activities Committee are charged with deciding where this year's class trip will be. You have a fixed budget for the class and you need to figure out what will be the most fun and affordable option. Your committee members have collected a bunch of brochures from various parks - e.g., Marine World, Great Adventure, and others (see inbox of materials) - which have different admissions costs and are different distances from school. You have also collected information about the costs of meals and buses. Your job is to plan and justify a trip that includes bus fare, admission and possibly rides, as well as lunch, within the fixed budget the class has.

Evaluate and recommend tasks: These tasks involve understanding a model of a situation and/or some data about it and making a recommendation. For example:

Safe driving distances

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DRAFT TO ACCOMPANY GOVERNING STATE VOTE ON ASSESSMENT CLAIMS
March 20, 2012
Posted by Oregon Dept. of Education, May 2012 - to be updated following future updates from Smarter Balanced
A car with good brakes can stop in a distance “D” feet that is related to its speed “v” miles per hour by the model: 
\[ D = 1.5vt + v^2/20 \]
where “t” is the driver’s reaction time in seconds.

Using this model, you have been asked to recommend how close behind the car ahead it is safe to drive (in feet) for various speeds of v miles per hour.

**Interpret and critique tasks:** These tasks involve interpreting some data and critiquing an argument based on it. Again the purpose of these tasks is not to provide a setting for the student to demonstrate a particular data analysis skill, but to draw conclusions in a realistic setting, using a range of techniques. For example:

**Choosing for the Regionals**

Our school has to select a girl for the long jump at the regional championship. Three girls are in contention. We have a school jump-off. Their results, in meters, are given below:

<table>
<thead>
<tr>
<th>Elsa</th>
<th>Ilse</th>
<th>Olga</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.25</td>
<td>3.55</td>
<td>3.67</td>
</tr>
<tr>
<td>3.95</td>
<td>3.88</td>
<td>3.78</td>
</tr>
<tr>
<td>4.28</td>
<td>3.61</td>
<td>3.92</td>
</tr>
<tr>
<td>2.95</td>
<td>3.97</td>
<td>3.62</td>
</tr>
<tr>
<td>3.66</td>
<td>3.75</td>
<td>3.85</td>
</tr>
<tr>
<td>3.81</td>
<td>3.59</td>
<td>3.73</td>
</tr>
</tbody>
</table>

Hans says, “Olga has the longest average. She should go to the championship.”

Do you think Hans is right? Is Olga the best choice? Explain your reasoning.

This is not a complete list; other types of task that fit the criteria above may well be included. A balanced mixture of these types will provide enough evidence for Claim #4.